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Estimating effective thermal conductivity of two-phase materials

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Abstract

In this paper, correlations are proposed to estimate the effective thermal conductivity of two-phase materials. For any α, Maxwell equation for $0.0 < c \leqslant 0.10$ and phase inverted Maxwell for $0.9 \leqslant c \leqslant 1$ are considered. For concentrations between 10% and 90%, and low α (<20), an equation based on the unit-cell approach (constant isotherms) is proposed. For $\alpha > 20$, three correlations are proposed based on field solution approach which includes three α ranges viz. medium ($20 \leqslant \alpha \leqslant 100$), high ($100 \leqslant \alpha \leqslant 1000$) and very high ($1000 < \alpha$). The predicted effective thermal conductivity of two-phase system is compared with well-established models. Comparison of the predicted values of the correlations with experimental results is also made. The predictions of effective thermal conductivity of two-phase materials match well with the experimental values.

Keywords: Effective thermal conductivity; Unit-cell approach; Phase inverted Maxwell; Constant isotherm

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1. Introduction

The problem of estimating the effective thermal conductivity of two-phase materials is a classical one, which has defied exact analytical solution till date. The importance of two-phase materials like ceramics, granular materials, emulsions and metal foams lies in their applications in high performance cryogenic insulations, packed beds, heterogeneous catalysts and catalytic reactors, composite materials, power generation and powder metallurgy. The Maxwell solution [1] is the starting point to find out the effective conductivity of two-phase systems, but it is valid only for very low concentration of the dispersed phase. Rayleigh [2] considering a cube with equal sized spheres at the corners, proposed a correlation. Bruggeman [3] proposed another model, to consider the properties of a composite medium with concentration greater than zero. Hashin–Shtrikman [4] proposed the most restrictive bounds for the value of effective thermal conductivity. Zehner–Schlunder [5] proposed another model with particles in contact with each other and they also considered the effect of secondary parameters. Series [6,7] and asymptotic [8] approaches for several regular arrays of spheres have been proposed. However, as was shown by Churchill [9] the effective thermal conductivity of dispersion is independent of the size distribution and arrangement of the spheres. For practical purposes, these parametric dependencies are generally negligible when theoretical solutions and experimental data are compared. Models that are a combination of the series and parallel-phase distribution were also suggested [10]. A numerical study for effective conductivity based on a model made up of spheres in cubic lattice has been carried out [11]. Modeling of two-phase thermal conductivity for random distribution of spheres in a continuum of different materials was carried out by Raghavan–Martin [12]. The model is essentially based on unit-cell approach, which incorporates constant-heat flux condition.

The primary parameters influencing the effective conductivity are conductivity ratio (α) and concentration (c). Now there is no satisfactory solution for all ranges of α

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Nomenclature

c	volume concentration of dispersed phase
h	variable height of inclusion in the unit cell
K	non-dimensional effective thermal conductivity $= K_{\text{eff}}/K_{\text{con}}$
K_{eff}	effective thermal conductivity of two-phase system (W/m K)
K_{dis}	thermal conductivity of dispersed phase (W/m K)
K_{con}	thermal conductivity of continuous phase (W/m K)
K_{exp}	experimental values of non-dimensional thermal conductivity
K_{pre}	estimated value of non-dimensional thermal conductivity by predicted correlations
$K_{\text{R–M}}$	estimated value of non-dimensional thermal conductivity based on Raghavan–Martin model
$K_{\text{Z–S}}$	estimated value of non-dimensional thermal conductivity based on Zehner–Schlunder model
K_{Brug}	estimated value of non-dimensional thermal conductivity based on Bruggeman model

Greek symbols

α	ratio of conductivities $= K_{\text{dis}}/K_{\text{con}}$, $0 < \alpha < \infty$
β	transformed ratio of conductivities $= (\alpha - 1)/(\alpha + 2)$, $-0.5 < \beta < 1$
BesselM[i,j]	modified Bessel function of order i and argument j

Subscripts

con	continuous phase
dis	dispersed phase
eff	effective

and c, because as the spherical particles come closer to each other, their fields interact producing higher order contribution which is difficult to model. In addition to the primary parameters there are the secondary effects influencing the effective conductivity such as contact resistance, radiation, convection and Knudsen effect and also parameters like particle size, shape, location, size distribution and orientation.

The above researchers have proposed models, which do not take into account the wide range of α and c. Recognizing the need for an engineering solution and realizing that a solution may lie in isolating higher order interactions from the Maxwell interactions; modeling of Maxwell interactions in the simple ways, a two-way approach to estimate the effective conductivity of binary mixture is adopted. In this paper a novel approach to predict the effective thermal conductivity of two-phase materials based on the unit-cell approach (constant isotherms) as well as on the semi-empirical field solution approach is proposed for wider range of two-phase systems. The model has been validated with the experimental data for different two-phase systems and it predicts effective thermal conductivity more accurately than the earlier proposed models.

2. Development of correlation for predicting an effective thermal conductivity

The unit cell with a particle inclusion height 'h' as the equivalent representation of the two-phase system is considered. Krischer [13] described the earliest model of the unit cube. The particle is represented as a cube with in large unit cube. The upper and lower limits to the conductivity of two-phase materials based on parallel and series resistance have given by Wiener [14]. Raghavan–Martin [12] developed a unit-cell model to calculate the conductivity based on parallel lines of heat flux. For systems, $\alpha < 20$, $0.1 \leqslant c \leqslant 0.9$, based on the resistance approach, the effective thermal conductivity, under the conditions of constant isotherms is given by

$$K = 1 + \frac{3\beta ch}{h + \beta[3c(1-h) - h]} \tag{1}$$

The inclusion height 'h' may be expressed in terms of K, β and c in Eq. (2)

$$h = \frac{3\beta c(K-1)}{3\beta c - (1 - 3\beta c - \beta)(K-1)} \tag{2}$$

'h' is estimated by averaging it over the two most restrictive bounds proposed by Hashin and Shtrikman [4] and is given by in Eq. (3)

$$h = \frac{\int_{K_{\min}}^{K_{\max}} \frac{3\beta c(K-1)}{3\beta c-(1-3\beta c-\beta)(K-1)}\,\mathrm{d}K}{(K_{\max} - K_{\min})} \tag{3}$$

where $K_{\max} = \frac{(1+2\beta)(1+2\beta c-\beta)}{(1-\beta)(1-\beta c+2\beta)}$ and $K_{\min} = \frac{1+2\beta c}{1-\beta c}$.

Integrating Eq. (3), we obtain

$$h(\beta, c) = \frac{-1}{\zeta} + \frac{1}{\zeta^2\Delta}\log\left[\frac{1-\zeta(K_{\min}-1)}{1-\zeta(K_{\max}-1)}\right] \tag{4}$$

where $\zeta = \frac{1-\beta-3\beta c}{3\beta c}$ and $\Delta = K_{\max} - K_{\min}$.

The effective conductivity in terms of ζ and h is given by

$$K = 1 + \frac{h}{1 + \zeta h} \tag{5}$$

The effective conductivity estimated from Eq. (5) shows large deviations from the actual values for higher values of α (>20) because of higher order effects and higher distortions of the flow lines. So, we propose a new correlation based on the field solution approach for higher values of α (>20), a new function $G(f, \alpha)$ is defined for predicting the effective conductivity. The function $G(f, \alpha)$ is defined in Eq. (6)

$$G(f,\alpha) = c^m\left\{f^x\alpha\left(\frac{\sqrt{2/f\alpha}\left(\mathrm{BesselK}[0,\sqrt{8/f\alpha}]+\mathrm{BesselK}[2,\sqrt{8/f\alpha}]\right)}{\mathrm{BesselK}[1,\sqrt{8/f\alpha}]}-1\right)\right\} \quad (6)$$

For the parameters m, f and x, the respective expressions are given in following paragraphs. Also, a limiting condition is defined as

$$y = \mathrm{Limit}_{c\to 1}[G(f,\alpha) - G(f,1)] \quad (7)$$

The effective conductivity for higher value of α is given by

$$K = 1 + \frac{\alpha}{y}[G(f,\alpha) - G(f,1)] \quad (8)$$

For the functions m, f and x, the best fitting parameters are used and expressions are assigned as follows:

For $20 \leqslant \alpha \leqslant 100$, $0.1 \leqslant c \leqslant 0.9$,
we have $m = 2$, $x = 1.12$ and $f = \Gamma(1 + 2c/1 - c)$
where $\Gamma = 1.5 - 0.03155\alpha + 0.000167\alpha^2$.
For $100 \leqslant \alpha \leqslant 1000$, $0.1 \leqslant c \leqslant 0.9$,
we have, $m = 2$, $x = 1 + 2^{\kappa}/(13 + \kappa)\kappa$ and
$\Gamma = 100/\alpha^2 5^{(\kappa-1)}$
where $\kappa = \alpha/100$
and for $\alpha \geqslant 1000$ and moderate concentrations of the dispersed phase,
we have, $m \to 0$, $x \to 1$, $f \to 0.141663$ and $K \to G(f,\alpha)$

The proposed correlation for the effective conductivity of binary mixtures incorporates the Maxwell and the phase-inverted Maxwell solution in the concentration regions $0.0 \leqslant c \leqslant 0.1$ and $0.9 \leqslant c \leqslant 1.0$, respectively. So, for any value of α, the correlations for the effective conductivity for the given concentration regions are expressed as

$$\text{For } 0.0 \leqslant c \leqslant 0.1, \quad K = \frac{1 + 2\beta c}{1 - \beta c} \quad (9)$$

$$\text{For } 0.9 \leqslant c \leqslant 1.0, \quad K = \frac{(1 + 2\beta)(1 - \beta + 2\beta c)}{(1 - \beta)(1 + 2\beta - \beta c)} \quad (10)$$

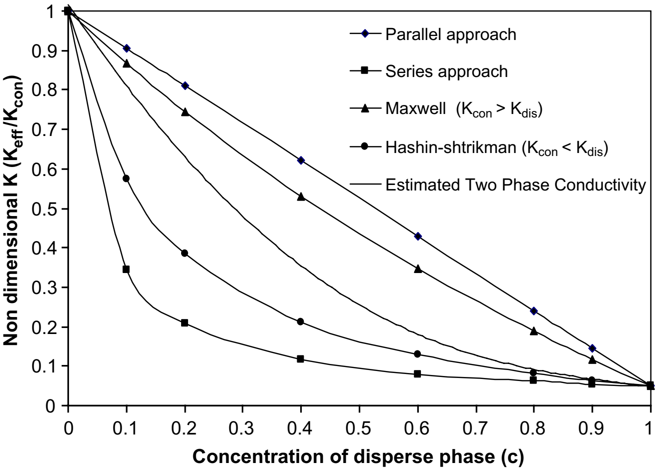

Fig. 1. Schematic representation of the conductivity K as a function of c for $\alpha < 1$.

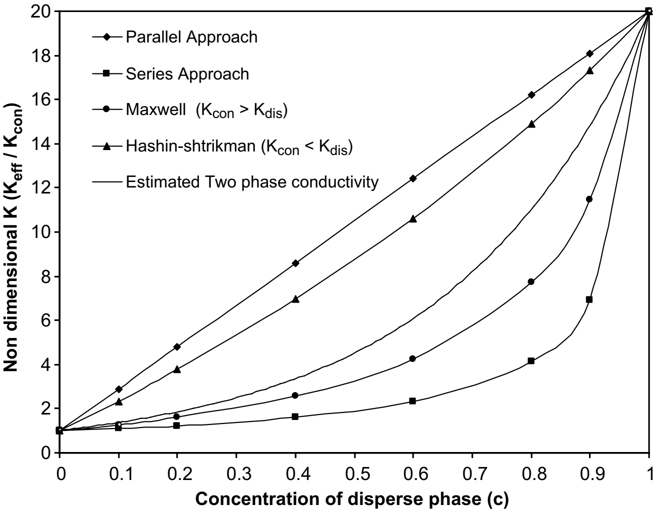

Fig. 2. Schematic representation of the conductivity K as a function of c for $\alpha > 1$.

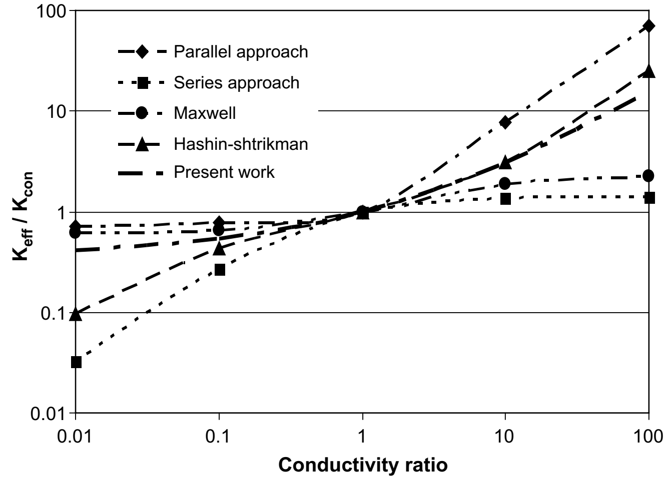

Fig. 3. Non-dimensional conductivity K as a function of conductivity ratio α for $c = 0.3$.

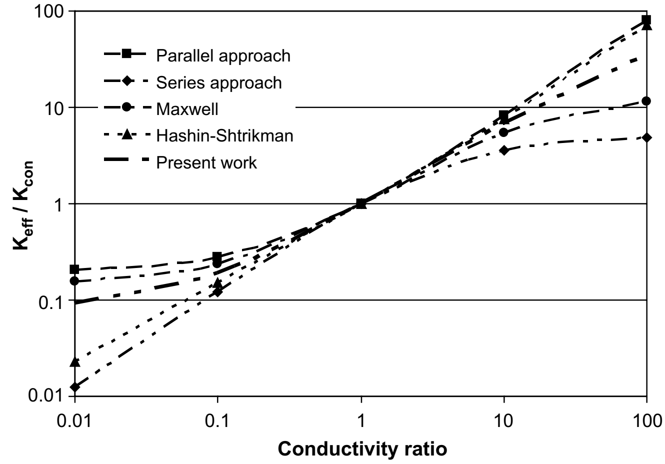

Fig. 4. Non-dimensional conductivity K as a function of conductivity ratio α for $c = 0.8$.

3. Comparison with other models

Non-dimensional conductivity of a two-phase system is mainly dependant on c and α. The variation of non-dimensional conductivity with concentration c for $\alpha < 1$ (0.05) is shown in Fig. 1. The two limits of Maxwell [1] and Hashin–Shtrikman [4] lines lie between the extreme limits of parallel and series [6,7] resistances, respectively, for the whole range of concentration varying from 0 to 1. The predicted conductivity of the material lies in between Maxwell and Hashin–Shtrikman lines for $\alpha = 0.05$. If the disperse medium conductivity is higher than the continuous medium $\alpha > 1$ then the variation of K will appear as in Fig. 2. The predicted conductivity curve was above the Maxwell curve for $\alpha = 20$ for all composition materials.

The variation of K with α for $c = 0.3$ and 0.8 are shown, respectively, in Figs. 3 and 4. The model has been com-

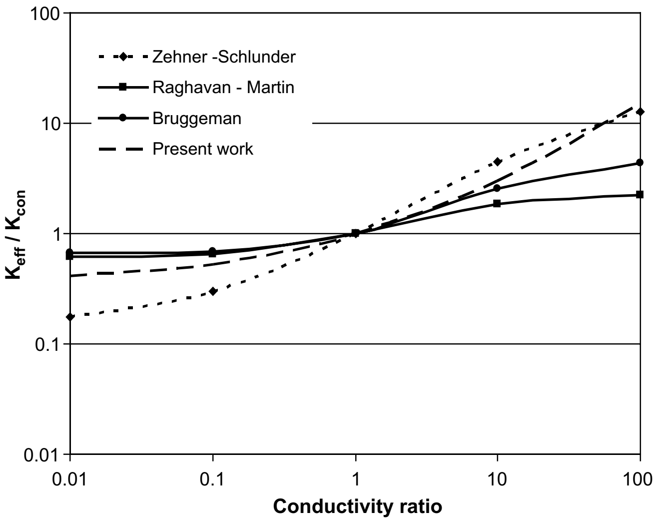

Fig. 5. Comparison of present work with the existing correlation for non-dimensional conductivity as a function of α for $c = 0.3$.

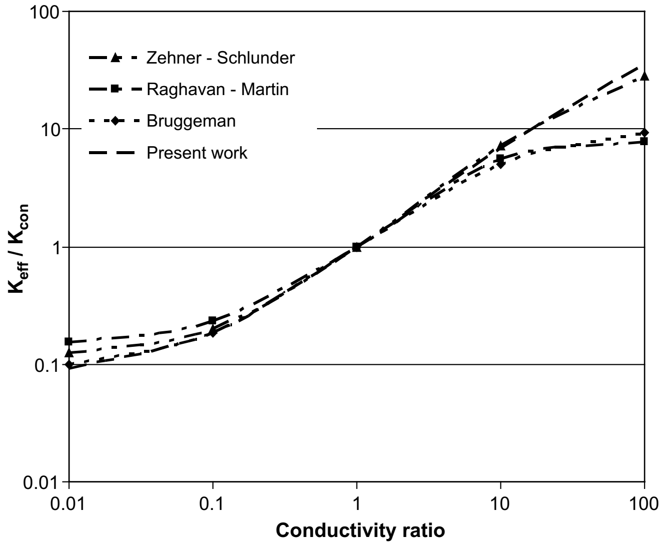

Fig. 6. Comparison of present work with the existing correlation for non-dimensional conductivity as a function of α for $c = 0.8$.

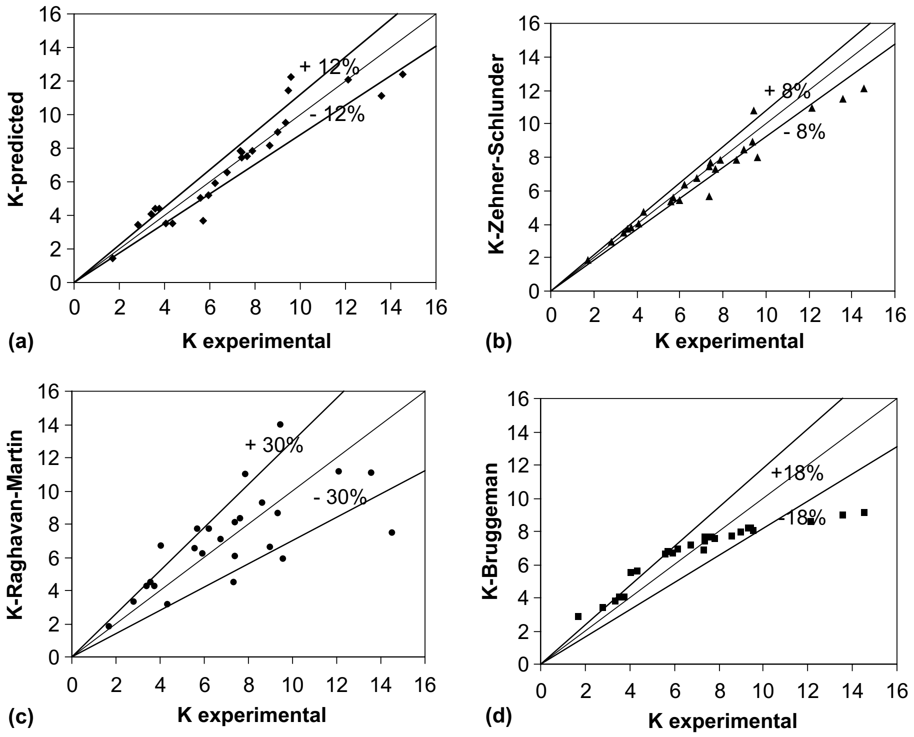

Fig. 7. Comparison of experimental results with (a) predicted work, (b) Zehner–Schlunder, (c) Raghavan–Martin, (d) Bruggeman equation for porous granular system.

pared with the standard models such as parallel, series [6,7], Maxwell [1] and Hashin–Shtrikman [4]. From the iso-conductance point, $\alpha = 1$, i.e., $\beta = 0$, the non-dimensional K goes to unity for all lines and also all the lines have the same slope. Since parallel and series resistances are correspondingly the upper and lower limits to effective conductivity of all type of two-phase materials; the present work line in between Maxwell and Hashin–Shtrikman

Table 1
Porous granular systems (solid phase/fluid phase)

Con. (c)	Cond. ratio (α)	K_{exp}	K_{pre}	K–R–M	K–Z–S	K–Bru	System	Ref.
0.74	45.79	9.458	**11.417**	13.95	10.81	8.1577	Glass sphere/air	[17]
0.20	45.79	1.708	**1.428**	1.835	1.871	2.844	Glass sphere/air	[17]
0.569	21.18	4.341	**3.501**	3.13	4.707	5.549	Silica sphere/water	[18]
0.495	61.91	5.9614	**5.21**	6.22	5.424	6.63	Stainsteel/Eth. alcohol	[18]
0.57	7.368	2.8194	**3.46**	3.27	2.96	3.371	Glass sphere/iso-octane	[18]
0.62	233.65	14.55	**12.375**	7.417	12.12	9.08	Lead shots/He	[19]
0.62	191.88	13.569	**11.102**	11.02	11.511	8.98	Lead shots/hydrogen	[19]
0.62	54.77	8.618	**8.1548**	9.22	7.811	7.701	Lead shots/water	[19]
0.47	95.285	5.714	**3.70**	7.71	5.593	6.768	Zircona powder/air	[20]
0.60	57.617	7.387	**7.737**	6.07	7.452	7.608	Lead/water	[21]
0.58	66.7	7.66	**7.499**	8.303	7.325	7.628	Zircona powder/air	[22]
0.64	66.7	9.36	**9.53**	8.61	8.94	8.142	Zircona powder/air	[22]
0.70	66.7	12.13	**12.09**	11.10	10.98	8.538	Zircona powder/air	[22]
0.65	42.89	7.857	**7.81**	10.98	7.848	7.526	Glass beads/air	[23]
0.65	8.578	3.571	**4.40**	4.507	3.73	4.01	Glassbeads/benzene	[23]
0.676	8.069	3.759	**4.42**	4.226	3.79	3.978	Quartz sand/water	[23]
0.60	37.62	6.206	**5.92**	7.67	6.407	6.9144	Glass beads/air	[24]
0.65	40.23	7.423	**7.463**	8.101	7.654	7.4116	Microbeads/air	[25]
0.639	7.864	3.398	**4.07**	4.205	3.484	3.77	Microbeads/soltrol	[25]
0.485	64.91	5.596	**5.04**	6.515	5.322	6.575	Wassau sand/*n*-heptane	[19]
0.64	56.96	9	**8.99**	6.58	8.45	7.916	Ottaw sand/helium	[19]
0.41	56.96	4.06	**3.54**	6.659	4.01	5.519	Wassau sand/helium	[19]
0.456	127.47	7.34	**7.84**	4.474	5.70	6.799	Miamisilt foam/air	[26]
0.552	127.47	9.6	**12.23**	5.899	8.03	8.008	Miamisilt foam/air	[26]
0.60	43.46	6.769	**6.55**	7.09	6.75	7.166	Glass/air	[27]

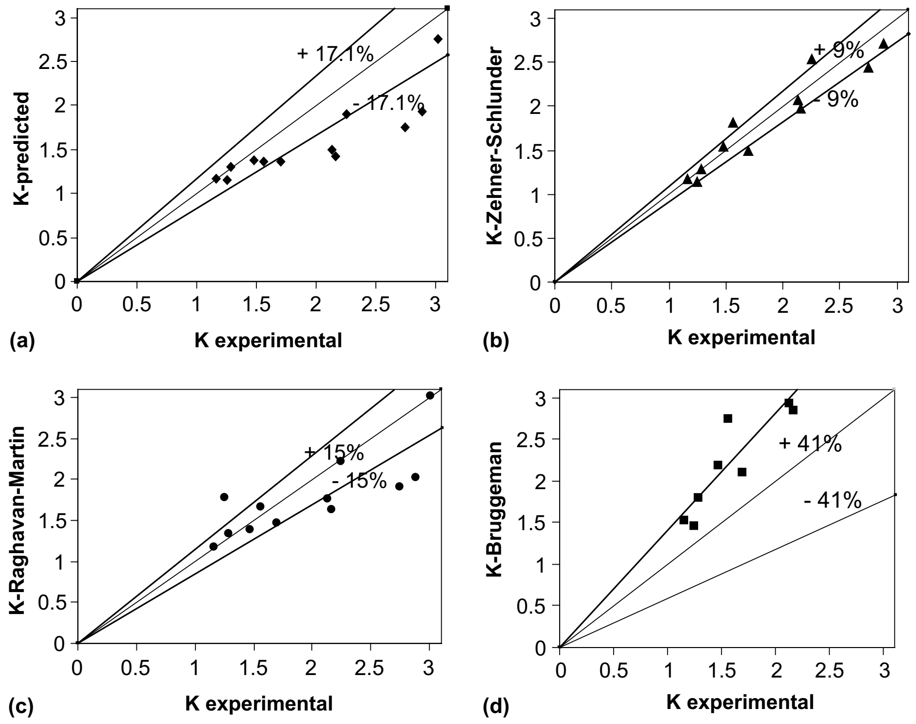

Fig. 8. Comparison of experimental results with (a) predicted work, (b) Zehner–Schlunder, (c) Raghavan–Martin, (d) Bruggeman equation for suspension system.

bounds for concentration 0.3 but the deviations from the present line come in to the plot for concentration increases to 0.8. This is because flow lines start to distorted when the concentration increases (higher order interaction) and also secondary effects come in to the system (contact resistance, convection and Knudsen effect).

The present work has also been compared with well established models such as Zehner–Schlunder [5], Raghavan–Martin [12] and Bruggeman [3] for concentration 0.3 and 0.8 (Figs. 5 and 6). For the higher values of conductivity ratio α, the Zehner–Schlunder model over estimates the K value and the Raghavan–Martin and Bruggeman results are under estimated the K value. The present model shows reasonably good trends for the concentration 0.3 and 0.8. For the lower limits of α all the models predicts well with the thermal conductivity data.

4. Comparison with experimental data

The present work has been tested it validity with published experimental data. The proposed correlations have depicted more accurately effective thermal conductivity of two-phase system. A comparison of the proposed correlations with the predictions of standard models like the Zehner–Schlunder [5], Raghavan–Martin [12] and Bruggeman [3] has been made for various two-phase systems. An effective thermal conductivity of porous–granular two-phase system has been estimated and compared with the other models. The parity plots for these systems are shown in Fig. 7(a)–(d). Estimation of K for various porous–granular materials with present work and other model is illustrated in Table 1. It is evident the present model has good agreement with experimental results. The range of accuracy

Table 2
Suspension systems (solid phase/liquid phase)

Con. (c)	Cond. ratio (α)	K_{exp}	K_{pre}	K–R–M	K–Z–S	K–Bru	System	Ref.
0.05	241	1.25	**1.155**	1.771	1.148	1.465	Graphite/water	[28]
0.11		1.7	**1.365**	1.468	1.493	2.109		
0.17		2.161	**1.42**	1.63	1.976	2.846		
0.24		2.887	**1.93**	2.022	2.717	3.815		
0.1	37.08	1.2857	**1.30**	1.33	1.292	1.80	Selenium/polypropylene glycol	[29]
0.2		1.564	**1.364**	1.66	1.82	2.75		
0.3		2.25	**1.904**	2.22	2.545	3.816		
0.4		3.0142	**2.76**	3.02	3.4933	4.92		
0.055	310.86	1.158	**1.172**	1.18	1.178	1.52	Aluminum/water	[30]
0.115		1.4748	**1.385**	1.381	1.550	2.179		
0.175		2.133	**1.50**	1.757	2.07	2.933		
0.21		2.748	**1.75**	1.909	2.442	3.41		

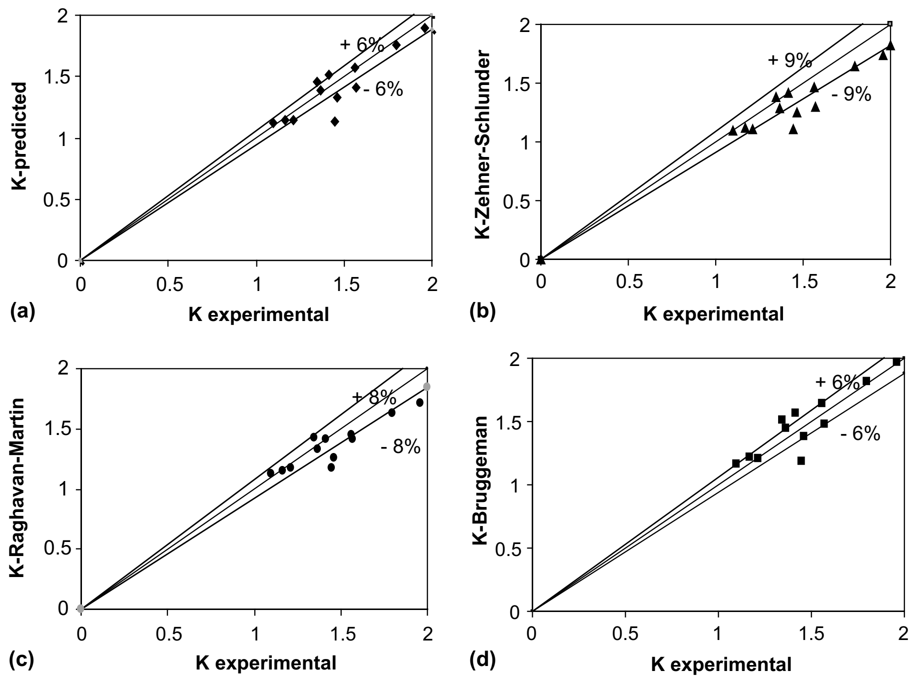

Fig. 9. Comparison of experimental results with (a) predicted work, (b) Zehner–Schlunder, (c) Raghavan–Martin, (d) Bruggeman equation for emulsion system.

appears quite good in consideration of the variety of source data selected with the wide range of shapes included. It is observed that an average deviation of only ±12% from experimental data as against 23.7% Raghavan–Martin [12], 8% Zehner–Schlunder [5] and 17.02% Bruggeman [3].

Similarly the comparison of other different types of two-phase systems (solid–solid mixtures, emulsion systems, suspension systems and granular porous systems) has been made for varying concentration values from 0 to 1 and for lower and higher values of conductivity ratio. The conductivity values of suspension systems (solid/liquid phase) are plot in Fig. 8(a)–(d). The predicted values are with in the range of 18% deviation from the experimental data (Table 2). All other models show a good agreement with experimental values in the range of 15% deviation except with Bruggeman model, because all the experimental values are of low concentration with higher conductivity ratio. The variation of predicted thermal conductivity of emulsion systems with that of experimental values is shown in Fig. 9(a)–(d). The emulsion systems have low concentration, therefore all the models estimated in the order of accuracy. From Table 3, it evident that for the emulsion systems, all the models predicts well with in 8% deviation from the experimental results because all experimental

Table 3
Emulsion systems (dispersed phase/continuous phase)

Con. (c)	Cond. ratio (α)	K_{exp}	K_{pre}	K–R–M	K–Z–S	K–Bru	System	Ref.
0.2	3.318	1.4615	**1.33**	1.262	1.253	1.376	Water/petroleum solvent	[31]
0.2	3.826	1.366	**1.386**	1.334	1.2853	1.442		
0.4	3.52	1.7976	**1.759**	1.627	1.643	1.816		
0.2	4.087	1.57	**1.414**	1.41	1.30	1.474	Water/mineral oil	
0.4	4.1	1.959	**1.897**	1.711	1.74	1.967		
0.1	3.313	1.445	**1.136**	1.168	1.105	1.186	Cellosize/flexol plasticizer	[32]
0.1	3.02	1.1	**1.125**	1.127	1.09	1.166		
0.3	3.313	1.415	**1.512**	1.4164	1.42	1.567		
0.3	3.021	1.347	**1.459**	1.429	1.383	1.5069		
0.1	3.66	1.213	**1.147**	1.171	1.113	1.208	Cellosize/polypropyline glycol	
0.1	3.72	1.168	**1.149**	1.153	1.114	1.212		
0.3	3.66	1.56	**1.572**	1.453	1.46	1.637		

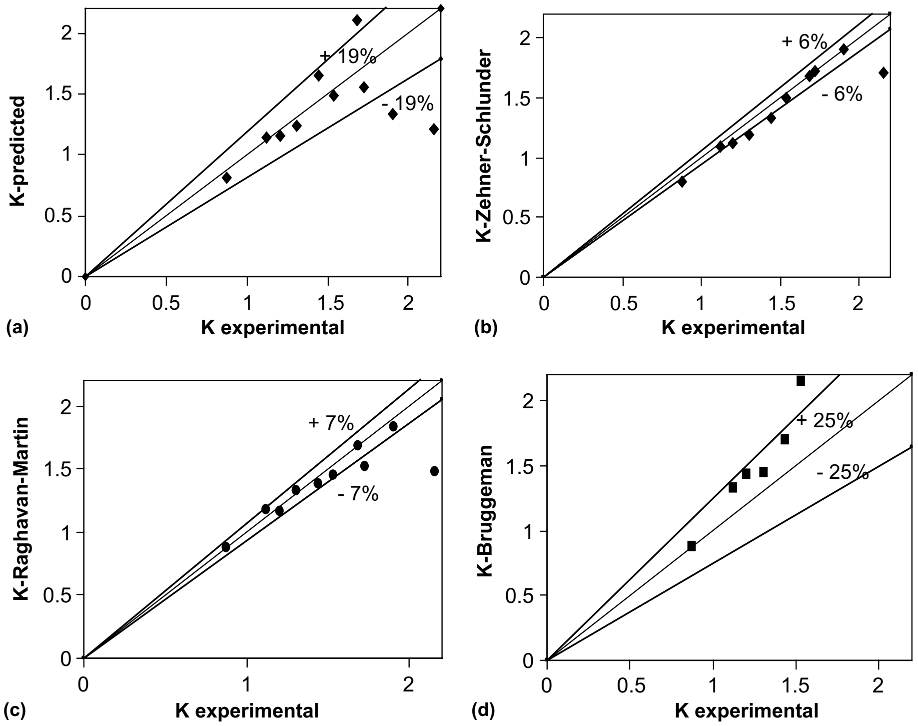

Fig. 10. Comparison of experimental results with (a) predicted work, (b) Zehner–Schlunder, (c) Raghavan–Martin, (d) Bruggeman equation for solid–solid mixtures.

Table 4
Solid–solid mixtures (disperse phase/matrix phase)

Con. (c)	Cond. ratio (α)	K_{exp}	K_{pre}	K–R–M	K–Z–S	K–Bru	System	Ref.
0.05	90.14	1.203	**1.152**	1.163	1.127	1.436	Lead powder/silicon rubber	[33]
0.16		1.726	**1.549**	1.511	1.7294	2.589		
0.05	21.63	1.125	**1.136**	1.175	1.094	1.327	Bismuth powder/silicon rubber	
0.16		1.536	**1.487**	1.455	1.498	2.142		
0.24		1.906	**1.33**	1.8287	1.907	2.803		
0.13	0.162	0.874	**0.816**	0.868	0.802	0.879	Forsterite/magnesia	[34]
0.15	132.33	2.16	**1.208**	1.48	1.715	2.531	Zincoxide/methylvinyl	[35]
0.10	9.517	1.3068	**1.24**	1.322	1.19	1.45	Silicapowder/dimethyl	
0.15	9.626	1.44	**1.65**	1.38	1.335	1.70	Silicapowder/methylvinyl	
0.25	9.626	1.6839	**2.10**	1.679	1.682	2.22	Silicapowder/methylvinyl	

values are low concentration and lower conductivity ratio, when compared to all the models our correlation is better with in 5.8% deviation from the experimental results. The comparison of solid–solid two-phase system is shown in Fig. 10(a)–(d). It is observed that the variation is about ±19%. The predictions with other models are illustrated in Table 4. The parity plots for granular two-phase systems are shown in Fig. 11(a)–(d). The predicted K values are illustrated in Table 5. The proposed correlations predicts better when compared to other three models, with in the range of 16% deviation.

The effective conductivity has also been estimated for two-phase system having medium and high concentration with high conductivity ratio (Fig. 12(a)–(d)). For medium and high conductivity ratio, only the proposed correlation predicts well (Table 6) with in 11% deviation from the experimental data. For higher concentration and higher conductivity ratio (Table 7) only our proposed correlation gives good agreement with the experimental results. Therefore the proposed model is good agreement for wide variety of two-phase systems.

5. Conclusions

The Maxwell solution and phase inverted Maxwell solution are applicable for lower ($c < 0.1$) concentration and higher concentration ($c > 0.9$), respectively, for any value of α. Three correlations have been developed to estimate the effective thermal conductivity. Eq. (5) based on the unit-cell approach (for constant isotherm) is applicable

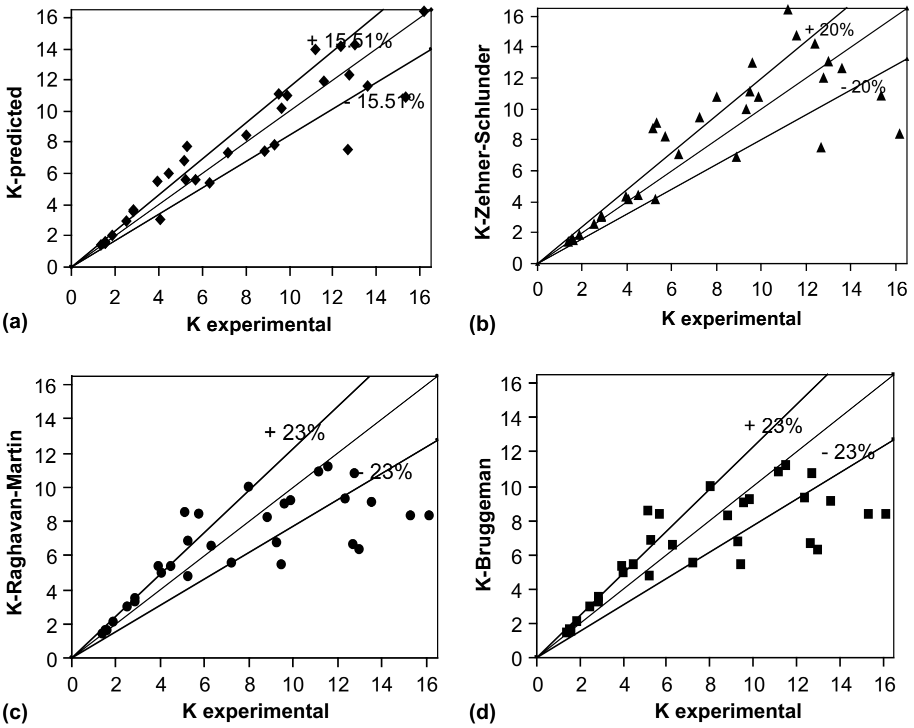

Fig. 11. Comparison of experimental results with (a) predicted work, (b) Zehner–Schlunder, (c) Raghavan–Martin, (d) Bruggeman equation for granular systems.

Table 5
Porous systems

Con. (*c*)	Cond. ratio (α)	K_{exp}	K_{pre}	K–R–M	K–Z–S	K–Brug	System	Ref.
0.507	128.6	8.879	**7.455**	8.2385	6.86	7.479	Air–calcite	[36]
0.511	684.6	9.295	**7.85**	6.701	9.97	8.186	Air–steel	[18]
0.511	138.1	6.328	**5.38**	6.507	7.087	7.58	He–steel	[18]
0.530	134.2	12.687	**7.561**	6.619	7.53	7.79	CO_2–basalt	[37]
0.535	19.7	4.051	**5.975**	4.974	4.18	5.152	Etoh–calcite	[38]
0.547	6.1	2.515	**2.96**	2.92	2.59	2.945	H_2O–calcite	[11]
0.550	1260.7	12.991	**14.289**	6.305	13.07	8.774	Air–lead	[36]
0.560	398.7	15.336	**10.90**	8.326	10.89	8.685	Air–quartz	[21]
0.561	17.9	3.963	**5.498**	5.292	4.31	5.143	H_2O–silica	[18]
0.563	16	5.244	**5.569**	4.78	4.13	4.92	Air–coal	[21]
0.563	2.2	1.524	**1.595**	1.615	1.55	1.603	H_2–coal	[21]
0.563	421.93	9.484	**11.126**	5.451	11.15	8.736	Air–silica	[18]
0.569	17.8689	4.494	**6.045**	5.378	4.39	8.736	Water–silica	[18]
0.569	7.648	2.859	**3.541**	3.471	3	3.438	IC8–glass	[18]
0.570	156.5	5.292	**7.70**	6.79	9.05	8.35	Oil–lead	[11]
0.571	745.19	9.615	**10.168**	9.038	12.94	8.954	Air–silica	[39]
0.571	1230.16	12.381	**14.205**	9.261	14.22	9.02	CO_2–SiC	[39]
0.572	2.03	1.5832	**1.53**	1.55	1.49	1.534	Glycerin–glass	[18]
0.575	129.71	5.146	**6.82**	8.524	8.766	8.271	He–SiC	[37]
0.575	104.37	5.724	**5.62**	8.372	8.24	8.08	H_2–SiC	[39]
0.576	583.33	13.583	**11.64**	9.139	12.58	8.96	CO_2–SiC	[39]
0.576	290.5	9.876	**10.97**	9.168	10.79	8.755	Air–SiO	[18]
0.577	3.023	1.891	**2.049**	2.107	1.88	1.97	Etoh–glass	[18]
0.580	7.824	2.862	**3.662**	3.265	3.10	3.52	He–glass	[18]
0.580	2.06	1.572	**1.551**	1.562	1.51	1.556	Glycerol–glass	[18]
0.580	1.812	1.384	**1.431**	1.428	1.41	1.435	H_2O–glass	[18]
0.600	124.2	7.213	**7.369**	5.536	9.47	8.483	Glycerin–lead	[37]
0.603	191.1	8.025	**8.475**	9.93	10.78	8.822	Etoh–lead	[18]
0.608	1130.5	11.181	**13.917**	10.8378	16.38	9.40	Etoh–Cu	[18]
0.612	253.3	12.775	**12.36**	10.758	12	9.056	Glycerin–Cu	[18]
0.613	608.5	11.560	**11.878**	11.203	14.76	9.35	H_2O–Cu	[18]
0.425	2277.1	16.154	**16.364**	8.309	8.39	6.996	Air–Fe	[18]

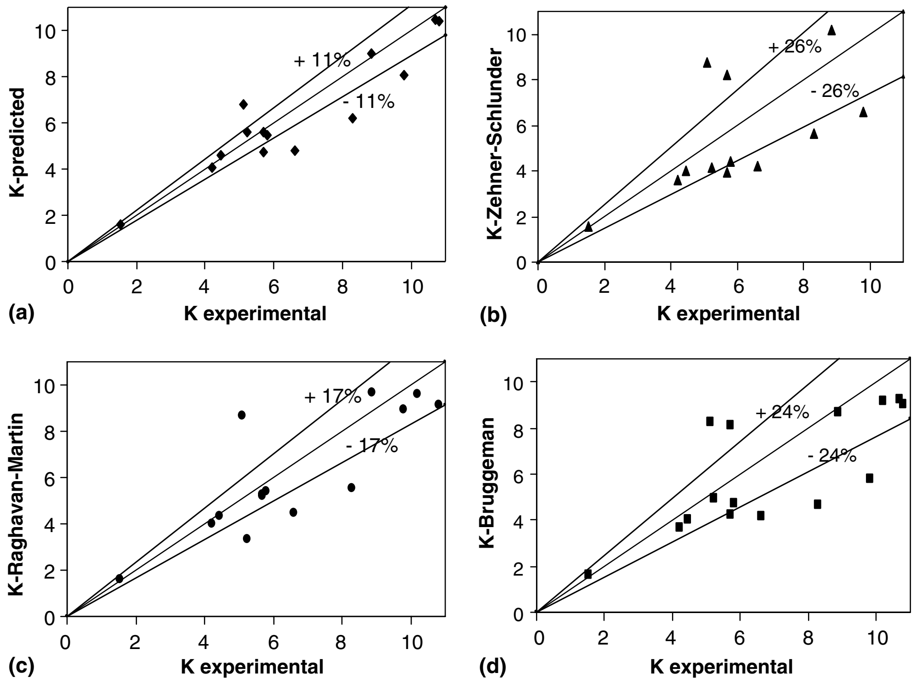

Fig. 12. Comparison of experimental results with (a) predicted work, (b) Zehner–Schlunder, (c) Raghavan–Martin, (d) Bruggeman equation for granular systems (medium concentration values).

Table 6
Porous systems (medium concentration values)

Con. (c)	Cond. ratio (α)	K_{exp}	K_{pre}	K–R–M	K–Z–S	K–Brug	System	Ref.
0.563	16	5.23	**5.57**	3.334	4.13	4.92	Air–coal	[11]
0.563	2.17	1.53	**1.58**	1.60	1.545	1.589	H_2–coal	[11]
0.575	104.4	5.7	**5.62**	5.28	8.25	8.08	H_2–SiC	[11]
0.575	743.8	10.8	**10.39**	9.16	13.158	8.998	Air–SiC	[11]
0.575	129.8	5.1	**6.825**	8.678	8.768	8.271	He–SiC	[11]
0.612	385.8	10.7	**10.46**	11.275	13.291	9.221	Cu–ethanol solution	[11]
0.612	253.3	12.8	**12.36**	10.853	12	9.056	Cu–glycerol solution	[11]
0.58	607.2	10.2	**11.76**	9.57	12.9	9.154	Air–MgO (100 C)	[11]
0.6	161.4	8.86	**9.03**	9.66	10.19	8.684	Air–sand	[10]
0.725	8.1	6.6	**4.833**	4.50	4.18	4.171	Behmite	[39]
0.866	8.1	8.3	**6.22**	5.512	5.625	4.633	Powder	[39]
0.77	14.5	9.8	**8.076**	8.96	6.569	5.781	Air/Ni/W catalyst	[11]
0.655	9.4	5.7	**4.71**	5.23	3.957	4.236	Air/Cr/Al catalyst	[11]
0.71	7.8	4.45	**4.58**	4.34	3.975	4.0296	Air/CO/MO catalyst	[11]
0.655	11.6	5.8	**5.456**	5.38	4.417	4.719	Air/SiO_2–Al_2O_3/catalyst	[11]
0.70	6.8	4.2	**4.077**	3.97	3.593	3.687	Air/Pt/Al_2O_3/catalyst	[11]

Table 7
Porous systems (high concentration values)

Con. (c)	Cond. ratio (α)	K_{exp}	K_{pre}	K–R–M	K–Z–S	K–Brug	System	Ref.
0.9	1167.3	992	**1000**	250	83	10.27	304 LSS powder	[40]
0.959	0.228	0.26	**0.243**	0.24	0.247	0.188	Ni alloy powder	[41]
0.945	0.071	0.095	**0.08**	0.1	0.099	0.017	Polystyrene foam	[42]
0.929	0.0302	0.0379	**0.036**	0.097	0.0665	−0.01	Glass foam	
0.925	1879.4	1606	**1676**	29.51	124.31	10.24	Epoxy resin foam	

for concentration varying from 0.10 to 0.9 and for lower α value ($\alpha \leqslant 20$). Eq. (8) based on the field solutions approach is applicable for concentration varying from 0.1 to 0.9 for various case such $20 \leqslant \alpha \leqslant 100$, $100 \leqslant \alpha \leqslant 1000$ and $1000 \leqslant \alpha$. The improved solution for higher concentration and higher conductivity ratio, predicts values in close agreement with experimental data for a larger range of α values. The results show that the present work superior to the other models for wide variety of two-phase system and the maximum deviation is found to be 11%. The predicted values of effective thermal conductivity show favorable agreement with the published experimental results.

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